MCS 220 Homework 2

1. Let F be a field and a, b, c elements in F. Prove the following statements.
   (a) The additive inverse of an element a is unique. That is if b and c are both additive inverses of a, then $b = c$.
   (b) If $a \neq 0$, then the multiplicative inverse of a is unique. That is if b and c are both multiplicative inverses of a, then $b = c$.
   (c) $0a = 0$.
   (d) $ab = 0$ if and only if $a = 0$ or $b = 0$.
   (e) $(-1)a = -a$. Hint: remember that $-a$ means the additive inverse of a.
   (f) $-(-a) = a$.
2. Let F be an ordered field and a, b, c elements in F. Prove the following statements.
   (a) If $a < 0$ then $-a > 0$.
   (b) If $a < b$ and $c < 0$ then $ac > bc$.
   (c) $0 \leq a^2$.
   (d) $0 < 1$.
   (e) If $0 < a$ then $0 < a^{-1}$ and if $a < 0$ then $a^{-1} < 0$.
   (f) $0 \leq |a|$.
   (g) $|ab| = |a||b|$.