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A MULTIFACETED APPROACH TO PROFESSIONAL DEVELOPMENT IN IMPERIAL COUNTY, CALIFORNIA

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STRIVE is an ongoing professional development project for beginner secondary mathematics teachers, which operates in one of the poorest and most educationally challenged parts of California. We take a multifaceted approach to training and supporting the participating teachers. We train them in mathematics content and pedagogy, observe and advise them, and have them engage in self-designed projects to improve their leadership skills. Our data show that this approach has resulted in significant increases in their mathematics content knowledge and increased their rate of retention in their jobs.
Professional development, mathematics teacher retention, mathematics content knowledge.

INTRODUCTION

The United States suffers from a perennial shortage of qualified mathematics teachers. Research into the causes of the shortage has found that the number of newly graduating mathematics teachers is more than sufficient to replace those that retire. The problem is that a large number of mathematics teacher leave the profession well before retirement age (Ingersoll & May, 2010). The attrition is particularly high in high-needs schools and among beginner teachers within the first few years of starting to work. There are various factors that can reduce the loss. Ingersoll & Perda (2010) found that the provision of content-focused professional development was one of the major predictors of mathematics teacher retention. In this paper, we are reporting on the results achieved by the Supporting Teacher Retention for Imperial Valley Educators (STRIVE) professional development project.

Imperial County is in the southeastern corner of the state of California along the U.S-Mexican border. One of the poorest counties in California, it has a population that is about 90% Hispanic and 42% functionally illiterate. Only 15% hold 4-year college degrees compared to the California average of 30%. The current unemployment rate of 28% is one of the highest anywhere in the United States. Imperial County schools consistently rank last in standard math achievement tests among California’s 58 counties, while California ranks number 47 among the 50 states. That this is an educationally underserved area is an understatement. A culture of low expectations prevails in the local schools. Since the area does not attract much outside talent, most of the teachers in these schools are graduates of the same schools. This sets up a cycle: teachers who went to underperforming schools return to teach at the same schools. There is a chronic shortage of qualified mathematics teachers. Like elsewhere in the USA, beginner math teachers often leave within the first few years in their positions. Some are let go because of concerns with their teaching effectiveness, others leave because they find the job overwhelming and are given little support to cope. Given the shortage of math teachers, these same teachers then reappear at other local schools. This kind of continual migration does not serve the cause of high quality math education.

It is in this context that the STRIVE grant operates. It is a five-year project. It runs 2007-2012 and provides professional development for beginner secondary math teachers—those within the first five years of teaching—in hopes of improving their mathematics teaching skills and their retention in their jobs. It is one of ten sites that are part of the Supporting Teachers to Increase Retention (STIR) master grant of the California Mathematics Project (CMP).

THE PROFESSIONAL DEVELOPMENT MODEL

STRIVE has taken a multipronged approach to the professional development of secondary mathematics teachers. We have run intensive summer institutes and academic year workshops, which covered both mathematics content and pedagogy, observed and coached teachers in their classrooms, sponsored a summer academy, where teachers could experiment in a low-stakes environment with the new content and pedagogy they learned, and supported teacher-based projects to develop leadership skills.

The importance of specialized mathematics content knowledge to teaching was highlighted by Ma (1999) in her influential book and by Ball and her collaborators in a number of articles (e.g. Ball et al., 2008). This type of content was a leading component of the professional development in STRIVE. As we already pointed out above, many of the participating teachers came to teaching mathematics via alternate pathways. In California, this usually means earning a college degree in some subject other than mathematics, typically liberal studies or engineering, and then passing the California Subject Examinations for Teachers (CSET) in mathematics at the appropriate level to demonstrate subject matter knowledge and/or completing a state-mandated number of units of mathematics courses. Unfortunately, such course work tends to consist of low-level courses. It is well known in mathematics education circles in California that preparing for and passing the CSETs is not comparable to earning a degree in mathematics. Thus, we have had a group with diverse mathematical backgrounds.

Taking a cue from Cuoco (Cuoco et al., 1996), our primary aim is to train the participants in good mathematical habits of mind, while we of course choose topics relevant to their daily teaching. In our experience, the biggest weaknesses in participating teachers' math content knowledge are in reasoning and complex problem solving. By complex problem solving, we mean solving problems that are not easily solved by one of the standard algorithms in the K-12 math curriculum, and require novel approaches. Those teachers that lack mathematics degrees normally would not have taken courses that include substantial reasoning and complex problem solving. Whereas those who did earn math degrees often compartmentalize and see reasoning and complex problem solving as limited to college-level math courses, and do not know how to include them in their own teaching. Therefore we made these the primary focuses of our mathematics content training in STRIVE.

Another element of the professional development in STRIVE is pedagogy/teaching strategies. Teachers learn about various approaches to teaching the content and engaging students with it, as well as activities and technologies linked to the math content that they could readily use in their classrooms. Examples include inquiry-based learning, mental math, guess-and-check tables, educational websites, and web-based math games. Since many of the teachers in STRIVE have been beginner teachers, who have not had years to accumulate teaching materials, worksheets, games, etc, they really appreciate this aspect of the training.

The effectiveness of professional development is strengthened when teachers have immediate opportunities to implement what they learn in their classrooms. For this reasons, three of STRIVE's intensive summer institutes were run in parallel to summer school. Participating teachers were observed in their summer classrooms by STRIVE staff and coached on their implementation of the content and pedagogy they were learning in STRIVE. In fact, in 2009, when severe cutbacks to educational budgets forced school districts in the Imperial Valley to scale down and often completely eliminate their summer schools, STRIVE funded its own two-and-a-half-week summer algebra academy for 7-8th grade students. There, participating teachers taught algebra in teams of two to small classes of junior high school students who were going to take algebra in their schools that fall. The teachers selected their own teaching objectives and designed their own curricula. They were observed and coached by STRIVE staff.

Finally, STRIVE funded teacher participants to engage in individualized academic year follow-up activities. In Years 1 and 2, they could choose any kind of activity that would improve their math content or pedagogical skills. The typical choices were taking mathematics or teacher education classes at SDSU-IV, or visiting and observing each other's classrooms. Starting in Year 3, teachers were asked to engage, either individually or in groups, in self-designed projects that would enhance and demonstrate leadership skills in addition to advancing math education in the Imperial Valley. Other than preparing participants to take the lead in doing something that mattered to them, the goal was to prepare them to be able to propose and seek out funding for such activities that went beyond their regular job duties, to empower them with a sense of ownership in their positions, and to increase their visibility and let them build closer work relations with their school and district administrators. For example, some teachers prepared student teams for math competitions or organized the competitions themselves, two teachers prepared a new financial math course for struggling students, and one teacher constructed a collaborative website where STRIVE participants could upload and share their instructional materials, and also started building a website to prepare Spanish-speaking students to be more successful at passing the California High School Exit Examination (CAHSEE).

Since mathematics content knowledge is one of the research topics we are reporting on in this paper, we would now like to give an idea of what the content in STRIVE looks like. We will describe a typical day in one of our summer institutes on numbers and operations, and a typical exercise from one of the academic year workshops that requires complex problem solving.

An example of rigorous mathematical reasoning in STRIVE

Having already dealt with integers and the basic algebraic operations (addition, subtraction, multiplication, and division) on the set integers, we introduced fractions. For motivation, we used the typical chocolate bar model. Then we defined a fraction to be an ordered pair of integers (a,b), where $b\neq 0$, normally written as a/b. Then the teachers were asked to consider the equivalence of fractions, still using the chocolate bar model as motivation. This posed quite a challenge since we had not invented the multiplication of fractions yet. But they did eventually come up with the definition using cross multiplication: $a/b=c/d$ if $ad=bc$. We explored how this definition allowed us to reduce and expand fractions. Still using the chocolate bar model, we discovered the standard inclusion map $\boldsymbol{Z}\rightarrow\boldsymbol{Q}$ given by $n\mapsto n/1$.

Then we considered the proper definition of addition of fractions. We started with the simplest examples of adding unit fractions with the same denominator, then moved on to ones whose numerators were different from one. The teachers quickly discovered that adding the numerators was the right choice for the definition. They verified that this was consistent with the inclusion of integers and the definition of addition on the set of integers. Finally, they quickly realized that using the equivalence of fractions defined earlier, adding fractions with different denominators required no additional definitions, and could in fact be done using the familiar algorithm.

Now they could verify that addition of fractions still had the usual properties of associativity, commutativity, 0 as an additive identity, and that every fraction had an additive inverse. Finally, we could define subtraction of fractions in terms adding the additive inverse.

Like many other math content topics in STRIVE, this day was an eye-opener for many of the teachers. Few of the junior high school teachers had seen such a construction of rational numbers and their addition and subtraction from scratch. Some of them could use the chocolate bar model to explain equivalent fractions, but this was the first time that they saw equivalence of fractions as an abstract definition motivated by, but not dictated by the chocolate bar model. The usual algorithm of adding fractions is something that haunts many a junior high school student as a meaningless procedure and gives a headache to many a secondary math teacher. Seeing it emerge as a result of our choice—an educated and carefully considered choice—rather than a memorized procedure, was a novel experience for many of the teachers. Perhaps even more impressive to them was seeing how those algebraic properties, such as commutativity and associativity, that they needed to teach, but otherwise did not really know what to do with, followed from our definitions.

As for the few math teachers with degrees in math, this exercise brought back dormant memories of abstract algebra classes and made a connection between the college-level math they once learned and the secondary school math they teach.

An example of complex problem solving in STRIVE

A typical problem whose solution defies the standard algorithms taught in K-12 education and requires creative thinking is the problem of the monkeys and the coconuts. This is a classic problem whose many variations are easy to find on the internet. Here is one we used:

> Five monkeys spend the day gathering coconuts. At the end of the day, they all go to sleep. During the night, one of the monkeys wakes up and divides the coconuts into five equal piles. He finds that there is one coconut left over, so he just eats it. Then he hides his share of the coconuts, and combines the other four piles into one. Finally, he goes back to sleep. Soon thereafter, another monkey wakes up and does exactly the same thing. This monkey also has one extra coconut left, which he eats. This repeats with each of the five monkeys. When the monkeys finally all wake up in the morning, they divide the coconuts into five equal piles and have one left. What's the smallest number of coconuts they must have gathered?

The typical initial approach of junior high school teachers is a guess-and-check table, which does not help much. They then try to work backwards, but that fails too. High school teachers try solving an equation, until they realize that they have two unknowns and only one equation:

$$\frac{1}{5}\left(\frac{4}{5}\left(\frac{4}{5}\left(\frac{4}{5}\left(\frac{4}{5}\left(\frac{4}{5}(x-1)-1\right)-1\right)-1\right)-1\right)-1\right)=y \qquad (1)$$

This is in fact a Diophantine equation. There are standard ways to solve it, but they are not part of the K-12 curriculum and typically do not show up in the standard courses required for a math degree. This gave us a chance to talk about divisibility, its definition, and invent a method of solving Diophantine equations using that definition. In fact, there is a shortcut that can solve this problem very quickly. It requires noticing that the problem would be greatly simplified if the monkeys had gathered four more coconuts. Then all this monkey business about eating the left over coconuts is unnecessary and the resulting Diophantine equation

$$\frac{4^5}{5^6}x = y \qquad (2)$$

has the fairly obvious minimal solution of $x=5^6$ and $y=4^5$.

The educational value of the exercise for the participating teachers is to realize that interesting mathematics is not about finding standard algorithms to solve standard problems—we have computers that do that much faster and more reliably than people—but to invent novel ways of solving problems by using creative thinking coupled with problem solving experience.

RESEARCH METHODS AND RESULTS

Mathematics content

To test for increases in math content knowledge, we used tests developed by the Learning Mathematics for Teaching (LMT) project at the University of Michigan (Hill et al., 2004).

These are carefully designed pre and post tests of specialized mathematics content knowledge, which were equated by the authors of the tests using a control group of teachers. That is even though the pre and the post tests are different, the scoring scheme is such that comparing scores on the two tests is meaningful, and an increase in the scores corresponds to increase in content knowledge. It adds to the validity of this measure that Hill et al. (2005) found that first and third grade teachers' math content knowledge as measured by these tests was significantly related to their students' achievement gains.

The tests cover various topics, such as number concepts and operations, and algebra, at the elementary and middle school levels. We used different tests in different years, roughly corresponding to the topics in our summer institutes and workshops. We used both elementary and middle school level tests. Although our teachers were secondary math teachers, they found the elementary tests challenging enough. We never aimed to cover material specifically for the test, but we did develop general specialized math content skills that should have been helpful in answering the types of questions on the tests. To give an idea of the questions on the LMT tests, we include a released item here.

32. Students in Mr. Carson's class were learning to verify the equivalence of expressions. He asked his class to explain why the expressions a – (b + c) and a – b – c are equivalent. Some of the answers given by students are listed below. Which of the following statements comes closest to explaining why a – (b + c) and a – b – c are equivalent? (Mark ONE answer.)

a) They're the same because we know that a – (b + c) doesn't equal a – b + c, so it must equal a – b – c.
b) They're equivalent because if you substitute in numbers, like a=10, b=2, and c=5, then you get 3 for both expressions.
c) They're equal because of the associative property. We know that a – (b + c) equals (a – b) – c which equals a – b – c.
d) They're equivalent because what you do to one side you must always do to the other.
e) They're the same because of the distributive property. Multiplying (b + c) by –1 produces –b – c.

Below are the graphs comparing the pre and post test scores. Participants are numbered on the horizontal axis and sorted in increasing order by pre test score. The scores themselves are in standard deviations from the mean of the control population that was used by the LMT team to equate the tests. We used a one-tailed paired t-test to test if the increase in the scores was statistically significant.

In Year 1, we had 16 teachers who took the tests. We used the Middle School Number Concepts and Operations test from 2005. The data show an average increase of 0.48 standard deviations from the pre to the post test, which is statistically significant ($p=0.0083$). It is encouraging to see that the increase is fairly uniform. Even though the group's mathematical background was diverse, participation in STRIVE had a positive impact on almost everyone.

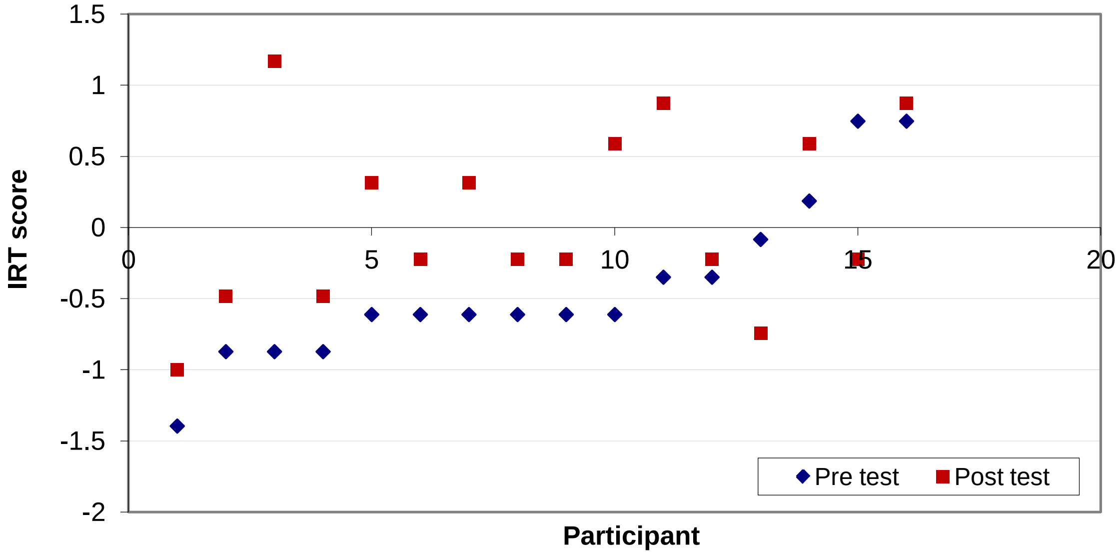

Figure 1. Year 1 pre and post test comparison

In Year 2, we had 22 teachers who took the tests. We used the Middle School Algebra test from 2007. The data show an average increase of 0.60 standard deviations from the pre to the post test, which is statistically significant (p=0.0004). The increase is again quite uniform.

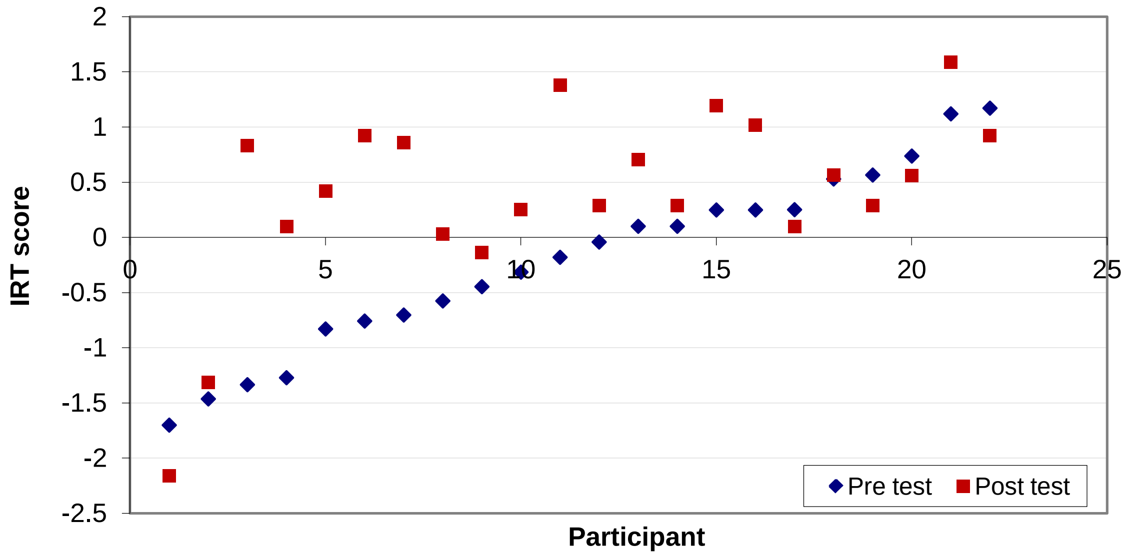

Figure 2. Year 2 pre and post test comparison

In Year 3, we had 20 teachers who took the tests. We used the Elementary School Algebra test from 2006. The data show an average increase of 0.65 standard deviations from the pre to the post test, which is statistically significant (p=0.0016). Once again, the increase was quite uniform.

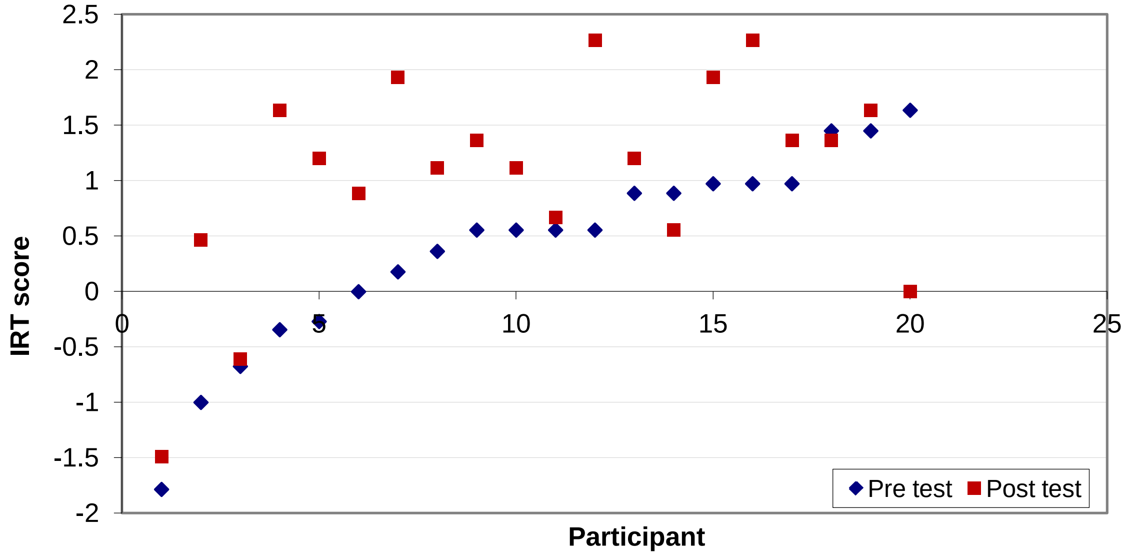

Figure 3. Year 3 pre and post test comparison

In Year 4, we had a new cohort of 7 teachers who took the tests. We used the online Middle School Number Concepts and Operations test. The data show an average increase of 0.10 standard deviations from the pre to the post test, which is not statistically significant (p=0.3000). It is harder to show statistically significant change in such a small population.

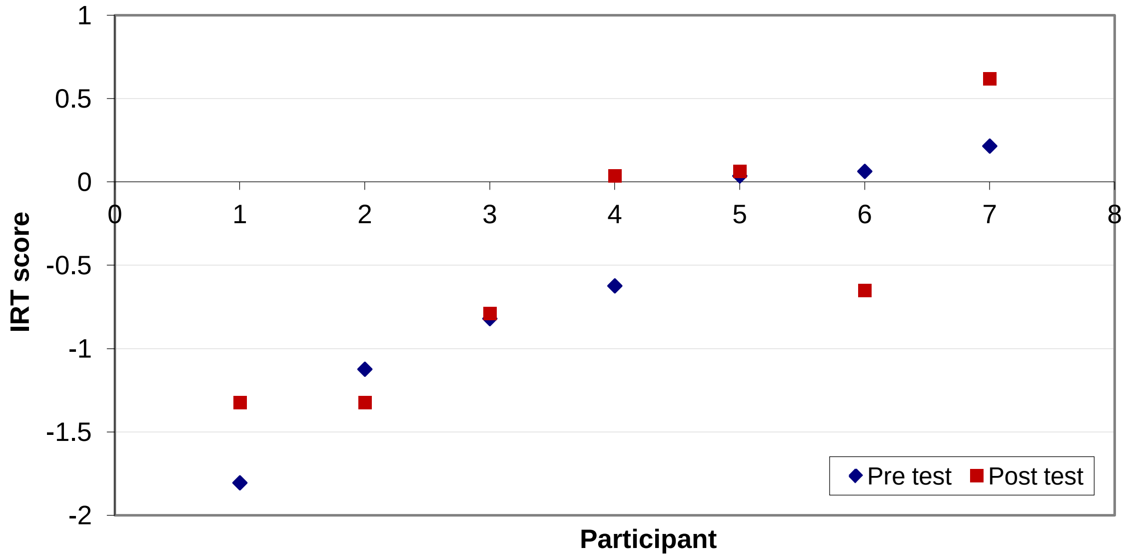

Figure 4. Year 4 pre and post test comparison

Retention

The other main goal of STRIVE was to increase teacher retention through professional development. We gathered pre-STRIVE retention data by following all secondary math teachers who had begun teaching in the Imperial County within the five years preceding the project (2002-2007). We computed the attrition rate for each school year by dividing the number of teachers who left teaching that year by the total number of teachers in the observed group who had begun teaching by then. Once the STRIVE project began, we used the same method to compute attrition rates, only we now observed the participants who were in STRIVE for at least a year. We continued to follow teachers even if they left the project. There were only two teachers in STRIVE we lost track of. We excluded them from this analysis.

Table 1: Attrition

School year	Attrition	Percent attrition
2002/03	1/8	12.5%
2003/04	1/13	7.7%
2004/05	1/18	11.1%
2005/06	4/26	15.4%
2006/07	6/32	18.8%
2007/08	1/11	9.1%
2008/09	1/23	4.3%
2009/10	1/24	4.2%
2010/11	1/32	3.1%

The significant decline in attrition starting in 2007/08, the year the STRIVE project began, is evident from the table above and the graph below.

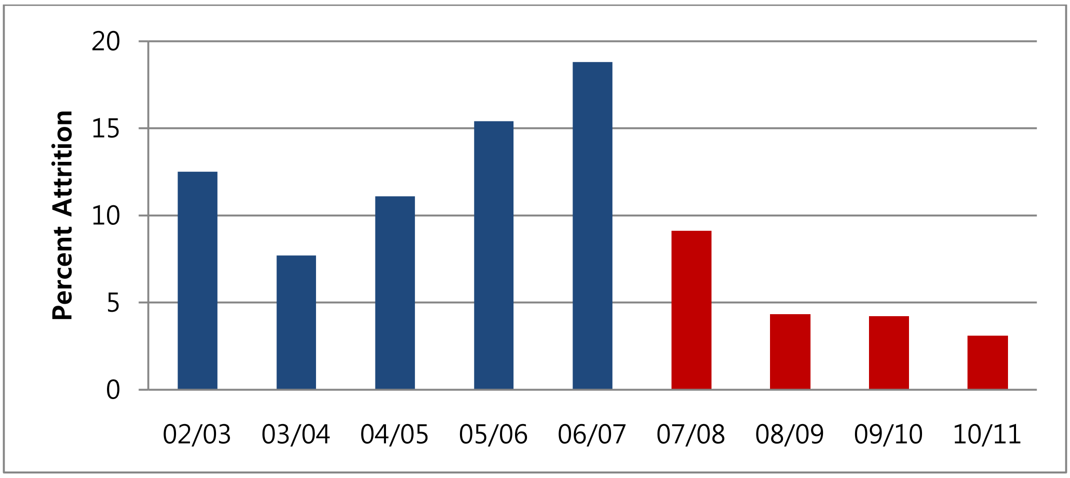

Figure 5. Annual attrition rate

CONCLUSIONS

We started our paper by describing the retention problem in K-12 mathematics education. We examined how appropriate professional development can alleviate this problem through our work with the STRIVE project. Our data show that our multifaceted approach to professional development resulted in increasing the rate of retention among participating teachers as compared to the rate before the STRIVE project. We did this by significantly increasing the specialized mathematics content knowledge of participants, as shown by their pre- and post test results on the LMT, involving them in leadership activities, and supporting them in their classrooms. Perhaps it is worth mentioning that in the process, we built a community of teachers who support each other and share ideas and educational resources with each other. In fact, such a community itself may be an important contributor to the increased rate of retention, although we have no research data to show that.

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